* Problems that should wait until later

Key

Special Focus: The Fundamental Theorem of Calculus

Multiple-Choice Questions on the Fundamental Theorem of Calculus

1. 1969 BC12

If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1} - e$ (D) $e^{-x^2} - 1$ (E) e^{-x^2}

2. 1969 BC22

If $f(x) = \int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, which of the following is FALSE?

(A) $f(0) = 0$ ✓

(B) f is continuous at x for all $x \geq 0$ ✓ differentiable $\forall x$

(C) $f(1) > 0$ ✓

(D) $f'(1) = \frac{1}{\sqrt{3}}$ $f'(x) = \frac{1}{\sqrt{x^3+2}}$ ✓

(E) $f(-1) > 0$

3. 1973 AB20

If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$

(B) $F'(b) - F'(a)$

(C) $F(a) - F(b)$

(D) $F(b) - F(a)$

(E) none of the above

$\int_a^b f(x)\,dx = F(b) - F(a)$

FTC

* **4. 1973 BC45**

Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \geq 0$. Which of the following statements is FALSE?

$g'(x) < 0$

(A) F takes on negative values. ✓

(B) F is continuous for all $x > 0$. ✓

(C) $F(x) = x\,g(x) - \int_0^x g(t)\,dt$ This is a by Parts justification

(D) $F'(x)$ exists for all $x > 0$.

(E) F is an increasing function.

Special Focus: The Fundamental Theorem of Calculus

5. 1985 AB42

$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

FTC - I

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$ (D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$

(E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

6. 1988 AB13

If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$ $f(c)-f(0)$

(A) $f(c)-f(0)$ (B) $|f(c)-f(0)|$ (C) $f(c)$ (D) $f(x)+c$ FTC

(E) $f''(c)-f''(0)$

7. 1988 AB25

For all $x>1$, if $f(x)=\int_1^x \frac{1}{t}\,dt$, then $f'(x)=$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x FTC

8. 1988 BC14

If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $\sqrt{1+x^3}$ FTC

(E) $\int_1^{x^2}\frac{3t^2}{2\sqrt{1+t^3}}\,dt$

9. 1993 AB41

$\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

FTC

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

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10. 1993 BC41

$f'(x) = 0$

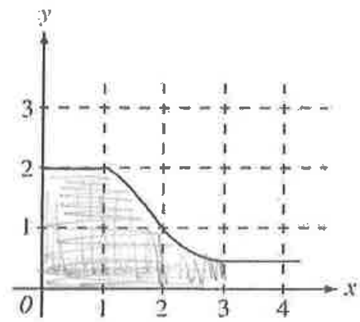
$f'(x) = 2x-3 \cdot e^{(x^2-3x)^2}$

Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ $\boxed{\text{(C) } \frac{3}{2}}$ (D) 2 (E) 3

$f'(x)$ ⊖ $\frac{3}{2}$ ⊕

11. 1997 AB78

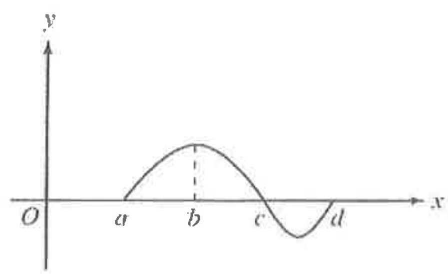


$\int_0^3 f(x)dx$

The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$ $\int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx = 2+2.3$

(A) 0.3 (B) 1.3 (C) 3.3 $\boxed{\text{(D) } 4.3}$ (E) 5.3

12. 1997 BC22



The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a (B) b $\boxed{\text{(C) } c}$ (D) d

(E) It cannot be determined from the information given.

Same <u>More</u> area

Special Focus: The Fundamental Theorem of Calculus

with Calc

13. 1997 BC88

Let $f(x)=\int_0^{x^2}\sin t\,dt$. At how many points in the closed interval $\left[0,\sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

$f'(x) = 2x\sin x^2$ $\quad$ $ARoc = \dfrac{f(\sqrt{\pi}) - f(0)}{\sqrt{\pi} - 0}$

$f(\sqrt{\pi}) = \int_0^{\pi} \sin t\,dt = 2$ $\quad$ $f(0)=0$ $\quad$ $ARoc = \dfrac{2}{\sqrt{\pi}}$

(Graph $2x\sin x^2$ and $\dfrac{2}{\sqrt{\pi}}$)

Calc

14. 1997 BC89

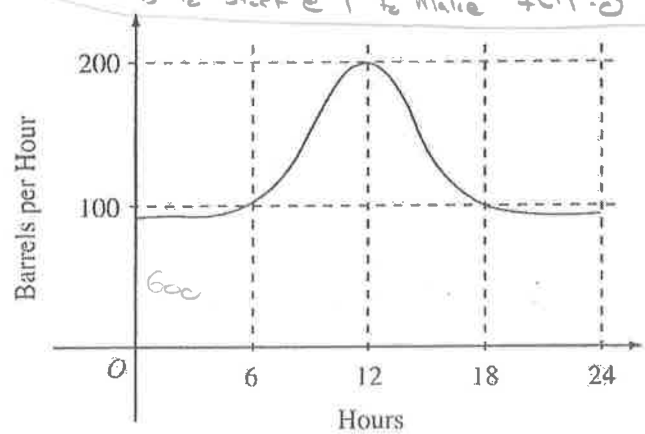
If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1)=0$, then $f(4)=$

(A) −0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

$f(x) = \int_1^x \dfrac{t^2}{1+t^5}\,dt$ then $f(4) = \int_1^4 \dfrac{t^2}{1+t^5}\,dt =$

x has to start @ 1 to make $f(1)=0$

15. 1998 AB9



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

Approximate area under curve

Special Focus: The Fundamental Theorem of Calculus

16. 1998 AB11

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

$f''(x)=0$ if $f(x)$ is linear

17. 1998 AB15

If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

$F'(x) = \sqrt{x^3+1}$; $F'(2) = 3$

(A) −3 (B) −2 (C) 2 (D) 3 (E) 18

18. 1998 AB88

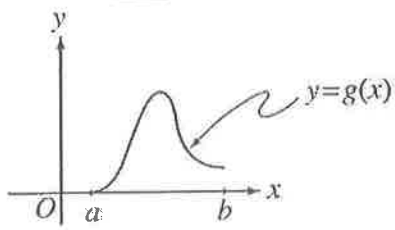
Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$ then $F(9) =$

$F(x) = \int_1^x \frac{(\ln t)^3}{x}$

$F(9) = \int_1^9 \frac{(\ln t)^3}{t}\,dt$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

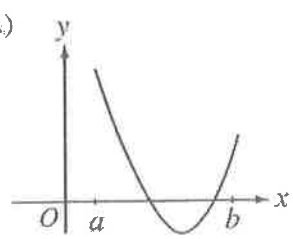
19. 1998 BC88



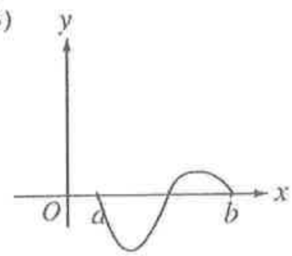
Value of integral getting bigger then taking away

Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

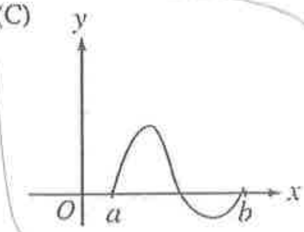
(A)



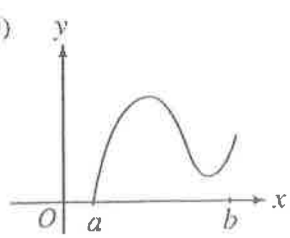
(B)



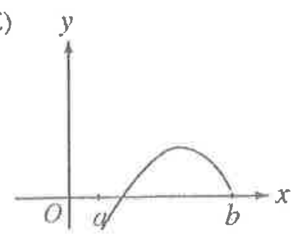
(C)



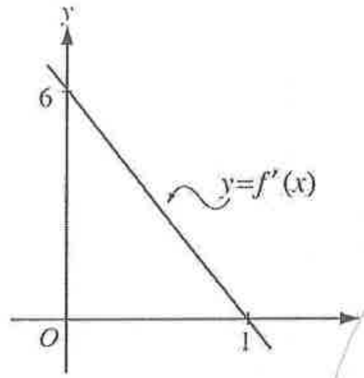
(D)



(E)



20. 2003 AB22



$f'(x) = -6x + 6$

$f(x) = -3x^2 + 6x + c$

$5 = 0 + 0 + c$

$f(x) = -3x^2 + 6x + 5$

$f(1) = -3 + 6 + 5$

$= 8$

The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 $\boxed{\text{(D) 8}}$ (E) 11

or $f(1) = f(0) + \int_0^1 f'(x)\,dx$

$= 5 + 3 = 8$

calc

21. 2003 AB82/BC82

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

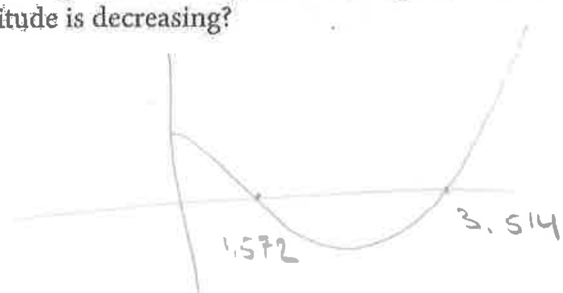
$\boxed{\text{(A) } \int_{1.572}^{3.514} r(t)\,dt}$

(B) $\int_0^8 r(t)\,dt$

(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_0^{2.667} r'(t)\,dt$



$\int_{1.572}^{3.514}$

Special Focus: The Fundamental Theorem of Calculus

Calc

27. 2003 BC80

Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

$\int_{7}^{14} \dfrac{100e^{-0.1t}}{2-e^{-3t}}\,dt$

$\boxed{\text{(A) 125}}$ (B) 100 (C) 88 (D) 50 (E) 12

Calc

28. 2003 BC87

A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

(A) 0.411 (B) 1.310 $\boxed{\text{(C) 2.816}}$ (D) 3.091 (E) 3.411

$s(0.655)$

Position $= 3 + \int_{0}^{0.655} v(t)$

Additional Problems

Skip

1. Suppose f is a continuous function such that

$$\int_0^x f(t)\,dt = x^2 \sin x - \int_0^x t^2 f(t)\,dt$$

for all x. Find an explicit formula for $f(x)$ and sketch a graph of $y = f(x)$.

2. Consider the function F defined by $F(x) = \int_0^x t\left|\sin(t)\right|dt$.

   (a) Compute $F'(x)$ and use this to find the intervals on which the graph of $y = f(x)$ is increasing and the intervals on which the graph is decreasing.

   (b) Find the absolute maximum value and the absolute minimum value of $y = F(x)$ (if they exist).

   (c) Sketch a graph of $y = F(x)$.

3. Consider the function F defined by $F(x) = \int_1^x (\sin t + \ln t)\,dt$ for $x > 0$.

   (a) Approximate the value(s) of x for which the graph of $y = F(x)$ has a local minimum value or local maximum value.

   (b) Sketch a graph of $y = f(x)$.

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

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with calc

a) $\int_9^{17} E(t)\,dt \approx 6004$ People

b) $15 \cdot \int_9^{17} E(t)\,dt + 11 \cdot \int_{17}^{23} E(t)\,dt$

$\approx$ \$104,048

c) $H'(t) = E(t) - L(t)$

$H'(17) = E(17) - L(17) = -380.281$

$H(17) = 3725$ means there were 3725 in the Park at $t=17$ (5:00 pm) and $H'(17) = -380.281$ means the rate of People in the Parks was decreasing at a rate of 380 People/hr

d) $H'(t) = 0 \Rightarrow E(t) = L(t)$

from graph $t = 15.795$

a) $g(-1) = \int_0^{-1} f(t)\,dt$

$= -\frac{1}{2}(3)(1) = -\frac{3}{2}$

$g'(x) = f(x)$

$g'(-1) = 0$

$g''(-1) = f'(-1) = 3$

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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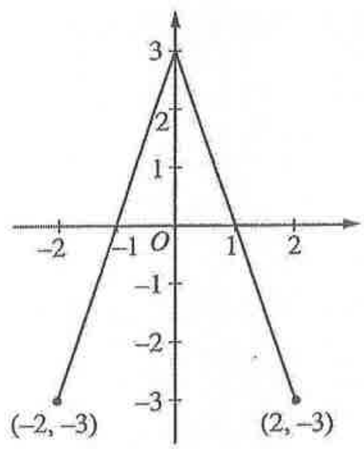
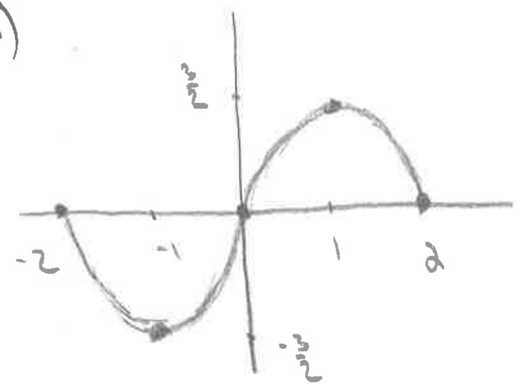
b) $g'(x) = f(x) > 0$ on $(-1, 1)$

c) $g''(x) = f'(x) < 0$ on $(0, 2)$



Graph of f

d)



4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)
